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# Interdependent Complex Consumer Residential Choice And The Oval Of Cassini

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## ABSTRACT

*This paper follows a theory of Nicosia and Hibshoosh regarding the choice by a social consumer unit who is facing conflicting institutional norms. The paper presents a duo-centric consumer residential choice model with special disutility restriction. This restriction captures the interdependence and complementary interaction in the effects that proximities to predetermined locations of competing institution yield on the disutility function. The properties of the Oval of Cassini play a key role in the parsimonious modeling of this phenomenon and in the analysis.*

*Specifically, we develop a residential consumer choice model where the consumer utility is affected by conflicting demands for activities of work and non-work institutions. The consumer unit is simultaneously attracted to two predetermined centers of work and non-work, while making its residence choice. We trace the consequences of these assumptions for optimal consumer choice of residential location, and for the size and price of property, level of a composite good, the level of internal conflict, taking into account externalities generated by population growth. The study identifies a preference for settling at the edges of a region along its main corridor in a two dimensional region. It also indicates a pattern of specifically directed curved regional growth in the periphery, with lesser development in the region's center.*

## INTRODUCTION

Costs of residing in the Bay Area region and commuting to work and non-work centers are high and their implications for industrial and regional planning attract close attention among planners and consumers alike. In a complex affluent society, consumers face time pressures as they attempt to perform conflicting work and non-work activities that are restricted by dictated norms of competing institutions (Hibshoosh & Nicosia (1987)). The development of marketing / economic models that recognize explicitly the interactive nature of the conflicting factors in the consumer utility would be helpful in development regional policy for land use and transportation.

Typically microeconomics based analytical urban economic models of residential choice and land-use adhere to the assumption of a single regional center. This center is assumed as the CBD ((Losch (1954), Burgess(1925), Alonso ( ( 1964),(1970)), Muth (1969), Mills (1972), Chan(2001)). The distance from the CBD center to the location of the customer residence represents the traveling distance to work. Non-work activities and locations of organizations associated with institutions administering them receive only a secondary attention in these models, and subsequently in land use and transportation planning. This often leads to a distorted planning perspective that supports mass transit development and dense regional development along few central transportation corridors. This comes at the expense of a less decentralized peripheral development of highway networks and residential development (Nelson, Niles and Hibshoosh (2001)). To provide a more balanced planning perspective, spatial models of residential choice for consumers attracted to multiple centers and thus encountering transportation costs to multiple centers have been developed. This paper belongs to this stream of research.

Within this stream of research, e.g., (Papageorgiou and Caseti(1971), Beckmann M.J (1976), Romanos (1977), Fujita and Ogawa (1982)), the approach to modeling the poly-center phenomena has been diversified and taken by the researchers in different directions reflecting alternate specific focus. However, we are not aware of any

modeling in this stream of research that poses an interactive complementary restriction on the proximity arguments of the consumer disutility. Instead, scholars have often modeled the utility specification in quite a general form, though they often considered the arguments of the spatial structure in the utility. When the general model was exemplified, the effects of the proximities to the centers were modeled typically as substitutes (additively) rather than as interacting complementarily. In our perspective, this left a substantive and modeling gap that we seek to address in this paper.

The purpose of the modeling is to enrich the conceptual base for regional planning by taking into account, in the utility function, the complex interactive pressure put on the individuals by the conflicting pulling institutions. We study the implications for consumer behavior and market evolution and development. What (geographical) patterns (continuous and discontinuous) of market evolution and residential choice could be expected? What effects do wealth and life style (frequency of visiting the different center) have on consumer residential choice? etc.

## **THE MODEL**

Conceptually, my modeling approach is based on the following fundamental results reached by Nicosia and Hibshoosh in our study of the institutional consumer, Hibshoosh and Nicosia (1987). In an affluent and complex society, the consumer is required to participate in variety of social institutions. In turn, the institutions issue distinctive norms that to some extent, sanction specific time demanding activities for their members. Examples of these institutions are: commercial shopping centers, schools, churches, arts and recreation centers, centers of political and community activities, etc. When the consumer unit is composed of several members as is often the case, an internal conflict within the consumer unit may naturally rise. The conflict is a result of contradicting commitment requirements of the different dominant centers of the different members. These are common cases in a family where husband or wife are working in different locations, where one member works and the second shops, and the work and shopping centers are located further apart, or where the child's school is located further away from the work place, etc. Institutional competition on the consumer's time results in an increased level of internal conflict that strongly affects the utility of the consumer unit. This conflict intensifies due to the interactive effect that norms of different institutions have on each other in a complex modern society where institutions are no longer relatively autonomous. The conflict also intensifies due to the partial replacement of the income constraint by time constraint in an affluent society. It is therefore reasonable to assume that the effect of accessibility to the organizations of different institutions, on the consumer disutility is also interactive. A distance or physical travel time to a center also represents psychological tension and costs that are generated by both a psychological distance that reduces potential intensity of activity and thought. A relatively closer proximity to institution may be associated with greater commitment to that institution and its norms. The norms of other institutions are competing and often conflicting. Hence, in our opinion, a greater degree of realism is obtained in the modeling, by assuming interaction among the proximity arguments in the consumer utility. I therefore choose to model the case where the effect of distances from the residential location to the work place and to centers of non-work activities is complementarily interactive and may be affected by network externalities. Hence, the following assumptions:

- A1** The region has only several centers common to all consumers. In the case of two centers: the work place center and the non-work center. The work center is common to all consumers and it is the main regional center (the CBD). The non-work center the (NWC) is also common to all consumers.
- A2** The consumer is a utility maximizer (or as common in spatial models a disutility minimizer).
- A3** Consumer disutility increases as the distance between the consumer residential location and each center increases. In the case of two centers, disutility increases as the distance between the residential location and the "work place" (i.e. the traditional C.B.D center) increases and as the distance between the residential location and the non-work center increases.
- A4** Reflecting complex consumer behavior, the effect of distances from the different centers is complementary. Thus in the case of two centers, the distance between the residential location and the work place center and the distance between the residential location and the non-work place center are assumed to be complementary factors in their effect on consumer disutility. They assume to create an intermediate state variable of conflict that consequently affects utility.
- A5** A consumer cannot reside in the immediate neighborhood of the centers. The centers are allotted to placement of the institutions. There is also a limit as to the maximum physical distance a consumer could

reside and still be considered a resident of the region. (In the marketing literature, the last 10 or 15 percent of customers who are furthest away from the retail locations are considered incidental rather than regular customers, outside the firm territory). Adopting this approach, a consumer who is located too far to utilize the region's institutions on a regular basis are considered non-residents.

- A6 The consumer budget is spent on the residence (including "service" cost of home at the chosen location), on transportation to the centers of social activity (work and non-work), and on other goods.
- A7 The transportation cost is proportional to the distance to the centers and is additive
- A8 Residence space is not infinitesimal. Hence, in any sub-region only a finite number of residents may live.
- A8 Regarding the price of land we make one of the following alternative assumptions:
  - a. The price of land is inversely related to transportation cost. i.e., it is inversely related to the sum of distances from the work center and the non-work center.
  - b. The base price of land is affected by population growth. The base price of land depends only on a uniform alternative use cost and transportation cost. However, as population grows, and bids for popular sites, the price of land at popular sites increases beyond the base price. The more popular the site the greater is the departure from the base price. However, the departure from the base price is smooth, with non-popular sites retaining their regular base.
  - c. The same arrangement as in b. with an additional possibility that the price of inferior locations drops beneath the base price, with a smooth departure that positively relates to site lack of popularity.
- A9 There are different consumer types (segments). Here we deal with segmentation based on lifestyle and wealth. Lifestyle is to be modeled by the frequency of visiting the center mix, wealth by disposable income.
- A10 An increase in the population has important impacts on center attractiveness due to the development of network externalities, and in turn on the pattern of market expansion, region density and cost of land.

Formal representation of the model is as follows:

Assume a plane region with two Focal points F1 and F2 representing centers of non-work activity and of work activity, respectively. The region is represented as a geometric plane. With a rectangular coordinate system, (X,Y), the location of each consumer's actual or potential residence is represented as a point (x,y) in (X,Y). The center of the region is the origin (0,0) and the focal centers F1 and F2 are located 2a units apart, equally distanced from the center's region, with coordinates (-a,0) and (0,a), respectively.

Every consumer unit in the region (typically a potential or actual household) has utility function U, of the type  $U = f(Y, c, W)$ , where the utility is derived from consumption of residential space of size W, from pooled proximity to the two centers (measured by an index c), and from Y, consumption of a composite of all other goods. The consumer object of choice is the mix of its location (residence), (x,y), together with W and Y. As a result, there are ultimately four decision variables. We denote the optimal solutions as  $x^*, y^*, Y^*, W^*$  and  $k^*$ . For  $(x^*, y^*)$ , the corresponding optimal distances to the centers are  $d1^*$  and  $d2^*$  respectively. It is important to notice that for any given pair  $(Y^*, W^*)$ , in the plane (X,Y), the "budget line" is an ellipse, with a fixed  $k^*$  and the indifference curves are Ovals of Cassini with c as an ordering parameter of the family of the indifference curves. In particular, at optimum, i.e., at  $(Y^*, W^*)$ , we obtain the ellipse with the parameter  $k^*$ , where  $k^*$  as a function of  $(x^*, y^*)$  is  $k^* = d1^* + d2^*$ . (in the more general case where n centers are present we can easily generalize to  $k = \sum d_i, i=1, \dots, n$ . in which case we obtain an n- ellipse. Further generalizations are similarly easily made.). Similarly, the optimal utility attained is evaluated at  $Y^*, W^*$  and  $c^*$  where  $c^* = d1^* d2^*$  the optimal oval of Cassini, is equivalent to the optimal indifference curve in the plane, (X,Y).

(In the more general case, we assume n centers and  $c = \prod d_i, i=1, \dots, n$ , thus we obtain the Cassini curve. Further generalizations are similarly easily made.)

Specifically, we assume that the distances  $d1$  and  $d2$  between the consumer residence and the two focal centers affect the utility in an interdependent complementary manner. Specifically, Hence, we assume  $c = d1 d2$ .

It should be noted that the effect of  $c$  on the utility is negative. It represents a disutility, namely, the higher is  $c$  the further the consumer is from the focal centers. Hence, we may also refer directly to  $c$  as a disutility index  $DU$  where  $DU = d_1 + d_2$ .

For ease of exposition, we may further assume that the functional form of  $U$  is of the following constant elasticity form,

$$U = Y^\eta c^\kappa W^\gamma \quad (1)$$

and thus,

$$\log U = \eta \log Y + \kappa \log c + \gamma \log W, \quad (2)$$

where  $\eta$  and  $\gamma$  are positive and  $\kappa$  is negative. Each consumer unit possesses a budget. The budget constraint is assumed as,

$$V = Y + t(d_1 + d_2) + rW \quad (3)$$

Where  $t$  is a transportation cost per distance unit and  $r$  the price (or rent) of residence per unit of land. Initially, we assume that  $r$ , the price of residential land at a location  $(x, y)$  is a function of the total travel distance,  $d_1 + d_2$ , and is inversely related to the total distance.

Formally,

$$r = g(d_1 + d_2), \quad (4)$$

where  $g$  is monotonically declining function in  $d_1 + d_2$ .

## ANALYSIS

### Effect of transportation on Demand for Residence Land and Composite of Other Goods:

To gain insights conveniently, we assume that  $g$  is linear in  $d_1 + d_2$ . Hence, the budget shape becomes,

$$V = Y + t(d_1 + d_2) + (a - (d_1 + d_2))W = Y + aW + (t - W)(d_1 + d_2) \quad (5)$$

And  $a - (d_1 + d_2) > m > 0$

Along any given ellipse  $d_1 + d_2 = k$ , the highest utility the highest consumer utility is obtained by choosing a location  $(x^*, y^*)$  that results in the pair of distances  $d_1^*$  and  $d_2^*$  that minimizes  $c$ , at  $c^*(k)$ . Hence, along any ellipse, given the conditionally optimal residence is selected, the problem becomes that of maximizing

$$\log U = \eta \log Y + \kappa \log c^*(k) + \gamma \log W \quad (6)$$

Or alternatively, since the second term is fixed, the maximization of

$$\log U^* = \eta \log Y + \gamma \log W \quad (7)$$

subject to

$$V^* = V - tk = Y + (a - k)W. \quad (8)$$

The solution to the later problem is,

$$Y_{opt} = \frac{\eta}{\eta + \gamma} V = \frac{\eta}{\eta + \gamma} (V - tk) \quad , \quad (9)$$

The further away the consumer unit is from the center, (a higher  $k$ ), the less resources it has available for consumption of other goods, due to increase in the transportation cost.

And

$$W_{opt} = \frac{\gamma}{(\eta + \gamma)(a - k)} V = \frac{\eta}{(\eta + \gamma)} \frac{V - tk}{(a - k)} \quad (10)$$

The effect of change in the distance as measured by moving to a further located ellipse is determined by the sign of the first derivative of optimal  $W$  with respect to  $k$ . This sign is identical to the sign of

$$\frac{-t(a - k) + (V - tk)}{(a - k)^2} = \frac{(V - ta)}{(a - k)^2} \quad (11)$$

The sign depends on the size of the product  $ta$  relative to  $V$ . If transportation cost is very high and the cost of land at center is also very high, a consumer residing further away would acquire smaller housing. This is because even further away from the center the cost of housing is still too high and the high cost of transportation reduces affordability. If the cost in the center not that high and transportation cost are less prohibitive, the size of housing further away from the center would increase.

Along the same ellipse  $d1 + d2$  is a fixed sum, and the budget constraint is linear in  $Y$  and  $W$ . Because  $t(d1+d2)$  is constant.

Consider the case where the level of land purchased is identical, say, a standard three-bedroom residence,  $W$ . With a fixed  $W$  we obtain,

$$(V - Y - aW)/(t - W) = d1 + d2.$$

And from (5)

$$V_k = t - W$$

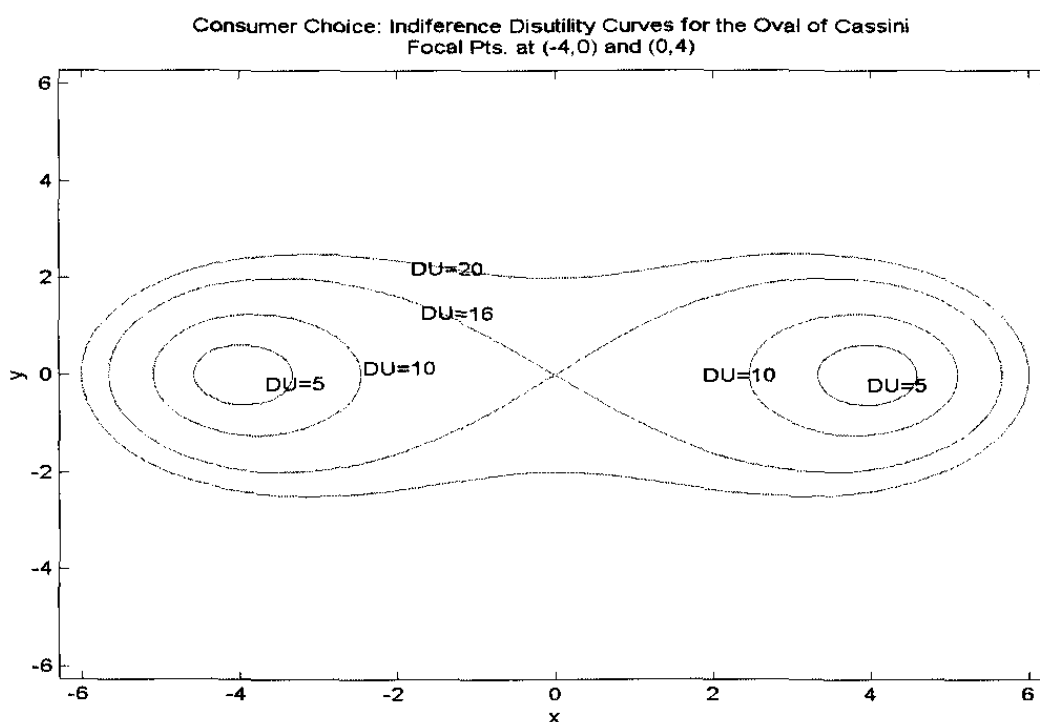
For a higher income for the same level of consumption of other goods and space, the consumer would be able to locate closer to the centers, in the sense that it will be on an ellipse closer to the center, assuming the  $W > t$ . i.e., the size of the marginal reduction in the rent of residence exceeds the transportation cost. If this is not the case and transportation cost exceeds the marginal reduction in the residential rent, the opposite is the case. The consumer unit would be able to locate further than the center, a choice that is not feasible at a lower budget. This may help explain why less affluent consumers often remain closer to the center, as they cannot afford higher transportation cost.

### Optimal residential location choice

We begin by observing the shape of the indifference curve for a given levels of  $Y$  and  $W$ . For a given  $Y$  and  $W$ , a given level  $U_0$  of  $U$  is obtained where the product  $d1d2 = C$ . Geometrically, the loci of the indifference curve for  $U_0$  in  $(x,y)$  is therefore the Oval of Cassini with a parameter  $C$ , (Fig.1), The indifference curves are densely ordered. For the same level of  $Y$  and  $W$ , a higher level of utility is identical to a location on an Oval of Cassini closer to the

origin. Optimizing means finding the point(s) (X\*,Y\*) where the highest oval of Cassini is tangent to the Budget line curve from within the Budget line. The first model considered the shape of the budget line is an ellipse. Hence, an optimization is identical with finding the location(s) where the Oval of Cassini is tangent to the Budget ellipse that circumscribes it. An oval that is circumscribed but not tangent within the cost ellipse cannot be paid for, and an oval which intersects the cost ellipse but is not tangent to is not optimal since a closer oval to the origin (in particular the tangent one to the ellipse) is attainable.

**Figure 1**



We will now prove that the tangency is obtained at (A,0) and (-A,0) where the cost ellipse intersects the horizontal axis X.

Any point on the cost ellipse with parameter k satisfies,

$$d1 + d2 = k. \quad (12)$$

$$d1 = \sqrt{(y^2 + (x - a)^2)} \quad (13)$$

$$d2 = \sqrt{(y^2 + (x + a)^2)} \quad (14)$$

$$k^2 = (d1 + d2)^2 = d1^2 + d2^2 + 2d1d2 \quad (15)$$

$$\begin{aligned} d1^2 + d2^2 &= y^2 + x^2 - 2xa + a^2 + y^2 + 2xa + a^2 \\ &= 2y^2 + 2x^2 + 2a^2. \end{aligned} \quad (16)$$

Hence,

$$d1d2 = .5k^2 - y^2 - x^2 - a^2 \quad (17)$$

Since  $k$  is a fixed minimum,  $d1d2$  is obtained where  $d1^2 + d2^2$  is maximized or alternatively, since the parameter  $a$  is a constant, where  $x^2 + y^2$  is maximized.

On an ellipse,

$$B^2 x^2 + A^2 y^2 = A^2 B^2 \quad (18)$$

$$y^2 = B^2 - \frac{B^2}{A^2} x^2 \quad (19)$$

and

$$x^2 + y^2 = B^2 + \frac{A^2 - B^2}{A^2} x^2 \quad (20)$$

Since  $A > B$ , it follows that  $x^2 + y^2$  is maximized where  $x$  is maximized. Maximum  $x$  on the ellipse is obtained at  $x = A$  and at  $x = -A$ . i.e., the Oval of Cassini with a minimal  $c = d1d2$  is tangent to any given ellipse that circumscribes it, at the intersection of the ellipse with the horizontal axis,  $(-A, 0)$  and  $(A, 0)$ .

The value of the index of the oval of Cassini at optimum is,

$$k_{opt} = \sqrt{(A - a)^2 (A + a)^2} = \sqrt{A^2 - a^2} \quad (21)$$

and the total travel distance to the focal points of work and non-work activities are,

$$c_{opt} = (A - a) + (A + a) = 2A \quad (22)$$

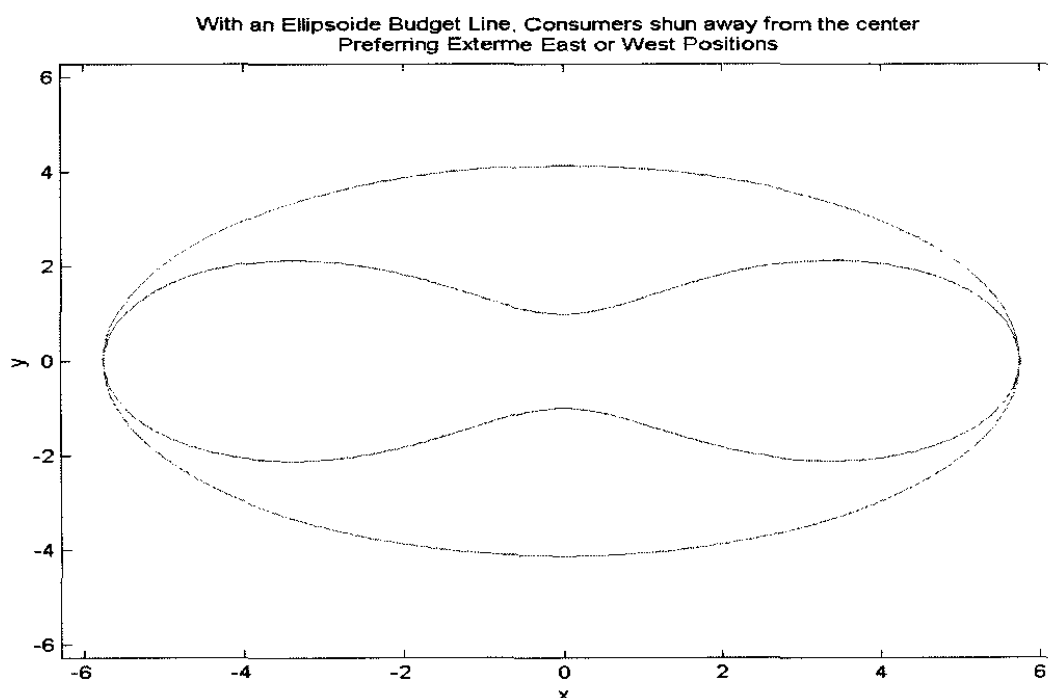
It then follows that the optimal location is at the edge of the markets. The optimal strategy of the consumer unit is to position itself closest to one center and furthest from the second center. So for example, the consumer unit would position itself closest to the workplace and furthest from the center of the non-work activity, or vice versa. This type of choice is the best strategy for a consumer unit in a complex affluent society with time constraints. (Hibshoosh and Nicosia (1987)). The consumer unit faces interdependent interactive and conflicting institutional directives and as a result possesses also interdependent and conflicting utilities or disutilities. By choosing to minimize one disutility while maximizing the other, the consumer unit minimizes the interactivity and interdependence effects in the conflict and maximizes its overall utility.

According to this result the customers positioning themselves along east and west of the centers and not in the hinterland between them. Affluent customers will be located closer to the centers and poorer ones further away from them.

Furthermore, this market will be a very thin one. There is a disincentive to reside outside of the direct route connecting the two centers.



Figure 2



### Effect of Population Growth

Although this is an extreme pattern, it does not appear totally unrealistic under the appropriate regional development assumptions and has immediate relevant implications for expected regional growth and regional development policy. In the early developmental stage of a region with an absence of external forces that introduce locational specialization and preferences for the centers, it is likely that all activities of work and non-work activities will be with located in a single center. However, locational specializations and preferences may be present for various reasons and may lead to two naturally separated centers, in our model of work and non-work activities. In the latter case, there will be pressure to link the two centers in a connecting road. The road will reduce travel-time, effective "distance" and transportation cost. As a public good it will increase the utility for all customers. The results of our model suggests that a proactive development of a horizontal road in the east west direction perhaps even prior to the settlement of new residents should be a recommended land use policy. Our conclusions are complementary to Hoyt's (1964) observations.

To this point, the treatment was constructed on the assumption of sparse population at the iso-cost strip. However, as a region grows many consumers are attracted to it; seeking residence. In this case, the higher the number of customers flocking to the region, the greater the competition for residential space.

How should we model this development? We assume that as population grows beyond some threshold level, relatively attractive locations are bidden up and require premium that is directly related to their attractiveness. Along any initial budget ellipse the least attractive points are the central one and the most attractive at the east and south edges.

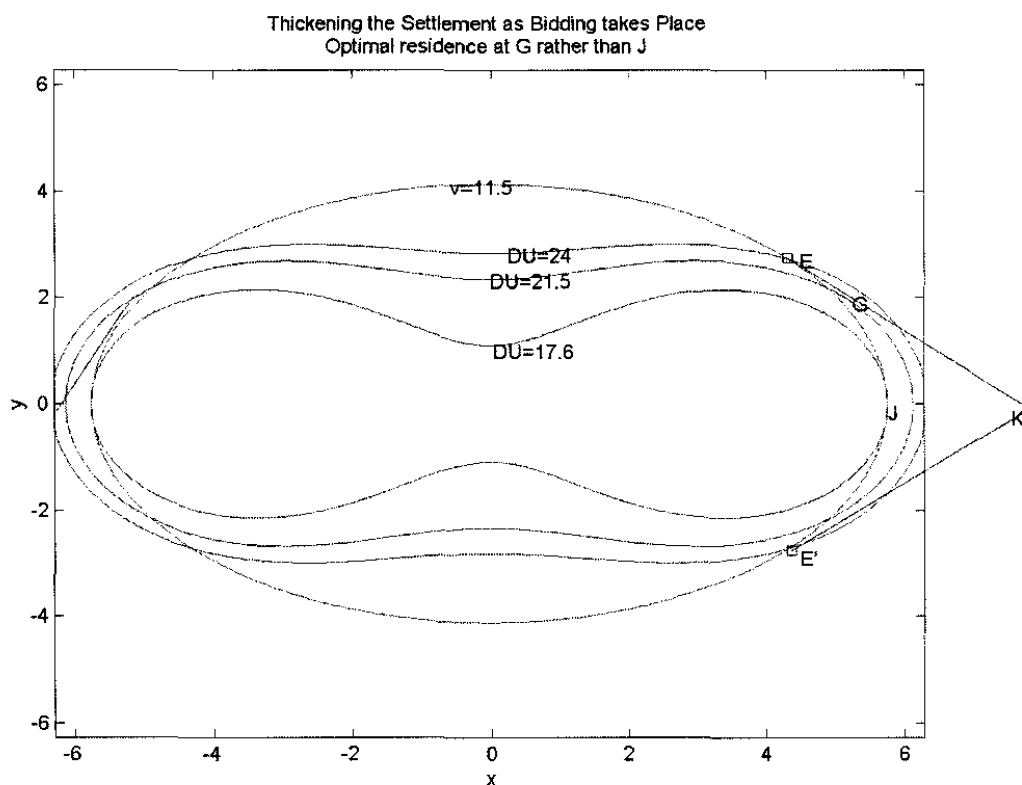
It is reasonable to assume that for some sufficiently distant ellipse from the center, there is a point,  $e$ , between the center and the edges where the threshold for gaining premium is just met. At any location more central than  $e$  along the same ellipse, the price of land is invariant and identical to that obtained with zero premium. However, from point  $e$  to and up to the nearest edge, land is sold at a premium that increases from zero at  $e$  to a maximum at the edge. As a result of this variable premium, the new budget line differs in part from the initial one

which applies where population externality was not substantially present. The change is depicted in figure 3, for the eastern side only, for the optimal budget curve for a given population level, where the externality effect is substantially manifested. Symmetrically similar changes that take place on the western side are not plotted nor discussed for simplicity. The new budget curve is identical with the old one in the center up till points  $e$  and  $e'$  (Symmetric points  $w$  and  $w'$  on the western side are not plotted). From points  $e$  and  $e'$  as it progresses in the eastern direction, the new budget line departs from the old one. It is here demonstrated for simplicity and without intended loss of generality as the straight segments  $ek$  and  $e'k$ . This budget line applies for a given  $Y_0, W_0$  that originally are a combination compatible with the ellipse budget line at a level  $k$ . In particular the budget applies for the case where  $Y^*$  and  $W^*$  are the optimal values of  $Y, W$ , in the presence of some level of population externality. In this case the optimal values for  $x$  and  $y$  are labeled  $x^*$  and  $y^*$  and the corresponding ellipse is identified by  $k^*$ . We will demonstrate that the point  $(x^*, y^*)$  is not located on the ellipse section of the budget line. The following discussion is for this optimal case. Compared with the old ellipse section of the budget line, the new section on the budget line  $ek$  is gradually increasing its horizontal gap from the old budget ellipse in the easterly direction, because residential properties closer to the strict east west directions are relatively more precious. The gap between the old and new line reaches its maximum in the strict eastern direction. Assuming the departure process from the old line is smooth, or alternatively that emerging new budget lines are smooth in the sense of possessing first derivative, the new sections of the budget line are tangent to the old section at points  $e$  and  $e'$ .

It also make sense to assume that the more distant is the budget ellipse from the center, the smaller is the pressure for location premium, as the associated land is less desired. Hence, at a more distant ellipse, point  $e$  is likely to be closer to the eastern ellipse edge, and the maximum gap of its new budget line from its eastern edge is expected to be smaller. When an ellipse budget line is located sufficiently far from the center, the population increase in the region is not significant enough to yield premium for distant properties, and the loci of the budget constraint remained invariant. As a result, the relevant budget strip will become more flattened than that at the initial ellipse. Since the edges of the ellipses are the most attractive point their price will be bidden up and the consumer would not be able to attain them with the Budget  $V$ . Instead, if the consumer would insist on residing along the  $X$  axis, the east west corridor, while using its old budget, it would have to settle at point,  $k$ , far away from edge of the market at  $J$ . At  $k$ , the utility is lower than at  $J$ . Notice,  $J$  is compatible with  $Y^*, W^*$ , as it is compatible with any pair  $(Y, W)$  that has originally the same  $V^*$ . We do not claim however that  $J$  is the one that will be obtained without population externality. In fact it is possible that the increase in the price of land will enhance the consumer to reside on ellipse that is further away from the center than the original one without population externality. i.e., the consumer will prefer to purchase less land and other goods and incur higher transportation cost as a result of the increase in the price of land. This lower amount of budget spent on land and other goods means corresponds under the original prices to an ellipse that is located further away from the origin. Notice, along  $ek$ , while  $(Y^*, W^*)$  is fixed, the rental price of  $W$  is changing. This is in contrast, with the ellipse section of the budget line, where the price of  $W$  is fixed, depending only on  $k^*$ .

However, utility maximization will not occur at  $k$ , but rather at some point  $G$ , that is internal to  $ek$ . This is because ellipses and ovals of Cassini cannot be tangent to each other at any point except for points at the strict eastern (or western) direction. Hence, the Oval of Cassini intersects and does not tangent to the old ellipse budget line at point  $e$ , as point  $e$  is not located in the strict east west direction. Since the new and old budget lines are tangent at  $e$  by the assumption of smooth changes in the budget line, tangency of the consumer indifference curve with the new budget line is impossible at  $e$ . Instead, the tangency at point  $G$  must be located to the right of point  $e$ . On the other hand the slope at point  $k$  of the segment  $ek$  is strictly negative, as more easterly properties command higher premium. Since the slope of any Oval of Cassini at any strictly eastern direction is infinite, an Oval of Cassini cannot be tangent to the new budget line at point  $k$ . Hence, the new consumer optimum at point  $G$  must be internal to the segment  $ek$ . The consumer optimal choice will be not to be located at the strict easterly direction, but rather to locate in a more peripheral and less extreme eastern position. Observe the following illustration (Fig.3). The initial budget  $V$  is  $v_1=11.5$  and an initial optimum is obtained at  $J$  with  $DU=17.6$ . However due to land bidding the budget constraint changes flattened and now  $EKE'$  replaces  $EJE'$  at a level of  $v=11.5$ . A location  $k$  is feasible, but not optimal. The disutility of locating at  $k$  is higher than at  $G$ .  $G$  is optimal with disutility of  $DU=21.5$ .

Figure 3



We can conclude that as population grows the region is filled and thickens from the central edges AA' toward the central periphery, rather than toward the region's center.

How does the center get filled? Notice that the settlement of the center is much less attractive due to institutional conflict. Hence, it would require either serious a reduction in price of land or a very high increase in alternative land in the region. The pattern suggests that it is possible that there will be divergence. Instead of settling in the center the consumer will decide to move to the periphery leaving the center relatively less populated with lower price value for land and with population who cannot afford a higher transportation cost. If this population is also of a lower socioeconomic class the center may continue to remain unattractive with lower property and land values. Notice however, that as population grows there is a greater pressure to filling the center.

#### Effect of differential preferences

Consumers differ in their locational preferences with some preferring a closer proximity to the work place rather than to the non-work place centers, and vice versa. We examined the effect by assuming interactive disutility with distinctive elasticities for proximities to the work center and the non-work center. Formally, we considered the following disutility model,

$$DU = d1^{\alpha} d2^{\beta} \quad (23)$$

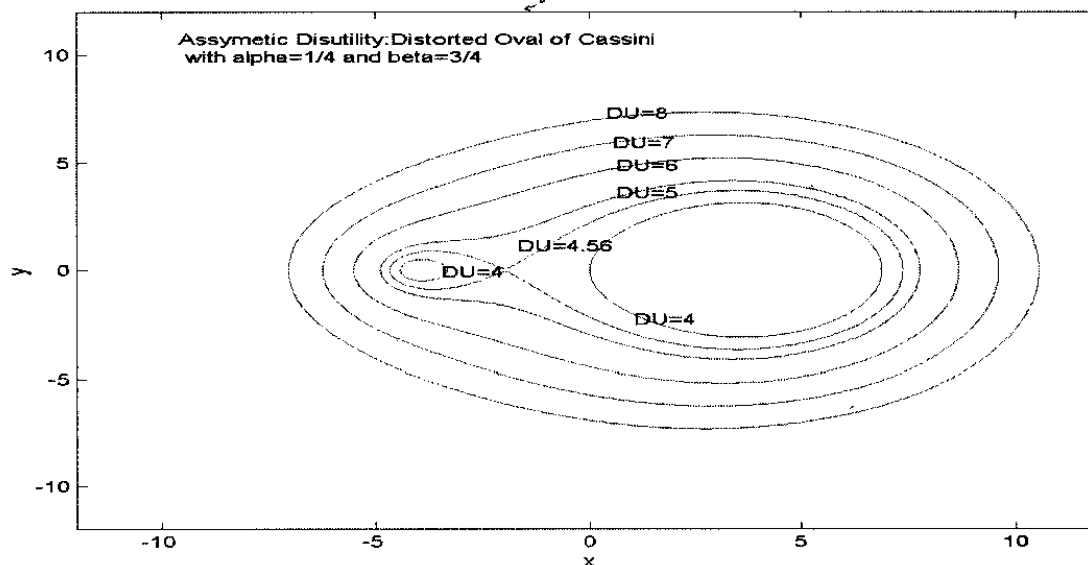
Although this form resembles a regular constant elasticity model of disutility, it is distinctively different, because of specific interdependency between  $d1$  and  $d2$ .

In terms of the coordinates of the residential location, the disutility is,

$$DU = ((x - a)^2 + y^2)^{\frac{\alpha}{2}} * ((x + a)^2 + y^2)^{\frac{\beta}{2}} \quad (24)$$

When  $\alpha=\beta$ , we have the case we analyzed so far with equal preference for a proximity to the work place and non-work place. A consumer with  $\alpha>\beta$  prefers proximity to the work place more than he prefers proximity to the non work place, and vice versa for a consumer with  $\alpha<\beta$ . In general, the map of the indifference curves of DU is similar to the one obtained in the case where  $\alpha=\beta$ , the maps of the Ovals of Cassini. In general, the indifference curve is a topological transformation of the symmetric indifference curve to a non-symmetric one, with a bloated segment around the preferred focal point (center), and a shrinking region around the less preferred focal point, Fig. 4).

Figure 4



Interestingly, the most important feature of the optimal location remains invariant when the budget strip is an ellipse. It remains the strictly East or West edge of the market. Except that in the non-symmetric case the consumer unit will settle only in the edge closer to its preferred center. The analysis where bidding takes place is also similar.

There is however some additional important difference. The map of the indifference curves shifts away from the non-preferred focal center toward the preferred focal center. As the bloated region around the focal point indicates, once bidding begins, a customer with a preference for proximity for a particular focal center (either work or non-work center), will be more ready to move to the periphery around that focal center rather than horizontally toward the region center (the origin). If most consumers share the same tendency, the trend toward thickening the settlement in the periphery around the preferred focal center will be expedited on the expense of filling the center of the region.

## CONCLUSIONS

Our results indicate that conflict among institutions demand may have important implications for consumer residential choice. According to our results a consumer will prefer to reside at the edge of the market in order to minimize disutility. This is a solution where ideally a consumer prefers a pure choice—being located closer either to the work center or to the non-work center and deliberately stays away from the region center. Consequently, the region center becomes neglected and its development lagging. Given this consumer preference, regional transportation planners should be engaged in mass transit along the main directions of the region. This recommendation for transportation policy is important because as population in the region grows, price of land along the preferred direction is rising and consumers must locate further away. Transportation development along the main route will reduce the pressure on the price of land.

The direction of regional residential development as population grows is also quite revealing. As prices along the main route are bidden up, consumers prefer to move to interior locations but away from the center of the

region. Consequently, this movement worsens the conditions at the center. It may lead to drop in price of land in the center reduction in the municipalities tax base and inadequate development. This situation may improve however when further population growth occurs and the cost of land in the center increases relative to the very far located residential alternatives. An increase in transportation cost would have a similar effect. The conclusion is that an optimal transportation policy should also focus on the development of a network of peripheral highways as the population in the region expands, even at a cost of higher taxes. At the same time regional planners should focus on increasing a center's attractiveness perhaps by offering variety of public goods and services at the region's center.

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